Thiele-Small based nonlinear model of loudspeakers

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Contents

1 Port-Hamiltonian System (`Core` object)

The Port-Hamiltonian structure in PyPHS is

$$\begin{pmatrix} \frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}t} \\ \mathbf{w} \\ \mathbf{y} \\ \mathbf{cy} \end{pmatrix} = \begin{pmatrix} \mathbf{M_{xx}} & \mathbf{M_{xw}} & \mathbf{M_{xy}} & \mathbf{M_{xcy}} \\ \mathbf{M_{wx}} & \mathbf{M_{ww}} & \mathbf{M_{wy}} & \mathbf{M_{wcy}} \\ \mathbf{M_{yx}} & \mathbf{M_{yw}} & \mathbf{M_{yy}} & \mathbf{M_{ycy}} \\ \mathbf{M_{cyx}} & \mathbf{M_{cyw}} & \mathbf{M_{cyy}} & \mathbf{M_{cycy}} \end{pmatrix} \cdot \begin{pmatrix} \nabla \mathrm{H}(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \\ \mathbf{cu} \end{pmatrix} \quad \text{with}$$

$$\underbrace{\begin{pmatrix} \mathbf{M_{xx}} & \mathbf{M_{xw}} & \mathbf{M_{xy}} & \mathbf{M_{xcy}} \\ \mathbf{M_{wx}} & \mathbf{M_{ww}} & \mathbf{M_{wy}} & \mathbf{M_{wcy}} \\ \mathbf{M_{yx}} & \mathbf{M_{yw}} & \mathbf{M_{yy}} & \mathbf{M_{ycy}} \\ \mathbf{M_{cyx}} & \mathbf{M_{cyw}} & \mathbf{M_{cyy}} & \mathbf{M_{cycy}} \end{pmatrix}}_{\mathbf{M}} = \underbrace{\begin{pmatrix} \mathbf{J_{xx}} & \mathbf{J_{xw}} & \mathbf{J_{xy}} & \mathbf{J_{xcy}} \\ -^{\intercal}\mathbf{J_{xw}} & \mathbf{J_{ww}} & \mathbf{J_{wy}} & \mathbf{J_{wcy}} \\ -^{\intercal}\mathbf{J_{xy}} & -^{\intercal}\mathbf{J_{wy}} & \mathbf{J_{yy}} & \mathbf{J_{ycy}} \\ -^{\intercal}\mathbf{J_{xcy}} & -^{\intercal}\mathbf{J_{wcy}} & -^{\intercal}\mathbf{J_{ycy}} & \mathbf{J_{cycy}} \end{pmatrix}}_{\mathbf{J}} - \underbrace{\begin{pmatrix} \mathbf{R_{xx}} & \mathbf{R_{xw}} & \mathbf{R_{xy}} & \mathbf{R_{xcy}} \\ {}^{\intercal}\mathbf{R_{xw}} & \mathbf{R_{ww}} & \mathbf{R_{wy}} & \mathbf{R_{wcy}} \\ {}^{\intercal}\mathbf{R_{xy}} & {}^{\intercal}\mathbf{R_{wy}} & \mathbf{R_{yy}} & \mathbf{R_{ycy}} \\ {}^{\intercal}\mathbf{R_{xcy}} & {}^{\intercal}\mathbf{R_{wcy}} & {}^{\intercal}\mathbf{R_{ycy}} & \mathbf{R_{cycy}} \end{pmatrix}}_{\mathbf{R}}$$

1.1 Dimensions

The system's dimensions are given below. Notice that a 0 value in the dimensions of the linear parts $\bullet_{\mathbf{l}} = \bullet - \bullet_{\mathbf{nl}}$ occurs if the system has not been split.

$\dim(\mathbf{l}) = n_{\mathbf{l}} = 3$

$\dim(\mathbf{n_l}) = n_{\mathbf{n_l}} = 0$

$\dim(\mathbf{x}) = n_{\mathbf{x}} = 3$

[1] `https://pyphs.github.io/pyphs/`

$\dim(\mathbf{x_l}) = n_{\mathbf{x_l}} = 3$

$\dim(\mathbf{x_{nl}}) = n_{\mathbf{x_{nl}}} = 0$

$\dim(\mathbf{w}) = n_{\mathbf{w}} = 0$

$\dim(\mathbf{w_l}) = n_{\mathbf{w_l}} = 0$

$\dim(\mathbf{w_{nl}}) = n_{\mathbf{w_{nl}}} = 0$

$\dim(\mathbf{y}) = n_{\mathbf{y}} = 1$

$\dim(\mathbf{p}) = n_{\mathbf{p}} = 0$

$\dim(\mathbf{o}) = n_{\mathbf{o}} = 0$

$\dim(\mathbf{cy}) = n_{\mathbf{cy}} = 0$

1.2 Constants

The system's constant substition values are given below.

parameter	value (SI)
L	0.011
R	5.7
K	40000000.0
M	0.019
A	0.406
i_{nvL}	90.90909090909092
i_{nvM}	52.631578947368425

1.3 Internal variables

The system's internal variables are given below.

- The *state* $\mathbf{x} : t \mapsto \mathbf{x}(t) \in \mathbb{R}^3$ associated with the system's energy storage:

$$\mathbf{x} = \begin{pmatrix} x_{\mathrm{L}} \\ x_{\mathrm{K}} \\ x_{\mathrm{M}} \end{pmatrix}.$$

- The *state increment* $\mathbf{d_x} : t \mapsto \mathbf{d_x}(t) \in \mathbb{R}^3$ that represents the numerical increment during a single simulation time-step:

$$\mathbf{d_x} = \begin{pmatrix} d_{\mathrm{xL}} \\ d_{\mathrm{xK}} \\ d_{\mathrm{xM}} \end{pmatrix}.$$

- The *dissipation variable* $\mathbf{w} : t \mapsto \mathbf{w}(t) \in \mathbb{R}^0$ associated with the system's energy dissipation:

  $\mathbf{w} = \text{Empty}.$

1.4 External variables

The controlled system's variables are given below.:

- the *input variable* $\mathbf{u} : t \mapsto \mathbf{u}(t) \in \mathbb{R}^1$ associated with the system's energy supply (sources):

  $\mathbf{u} = \begin{pmatrix} v_1 \end{pmatrix}.$

- the *parameters* $\mathbf{p} : t \mapsto \mathbf{p}(t) \in \mathbb{R}^0$ associated with variable system parameters:

  $\mathbf{p} = \text{Empty}.$

1.5 Output variables

The output (*i.e.* observed quantities) are:

- The *output variable* $\mathbf{y} : t \mapsto \mathbf{y}(t) \in \mathbb{R}^1$ associated with the system's energy supply (sources):

  $\mathbf{y} = \begin{pmatrix} i_1 \end{pmatrix}.$

  $i_1 = i_{\mathrm{nvL}} \cdot x_{\mathrm{L}}.$

- The *observer* $\mathbf{o} : t \mapsto \mathbf{o}(t) \in \mathbb{R}^0$ associated with functions of the above quantities:

  $\mathbf{o} = \text{Empty}.$

1.6 Connectors

The inputs and ouputs intended to be connected are given below.

- The *connected inputs* $\mathbf{u}_c : t \mapsto \mathbf{u}_c(t) \in \mathbb{R}^0$

  $\mathbf{u}_c = \text{Empty}.$

- The *connected outputs* $\mathbf{y}_c : t \mapsto \mathbf{y}_c(t) \in \mathbb{R}^0$

  $\mathbf{y}_c = \text{Empty}$.

1.7 Constitutive relations

1.7.1 Storage

The system's *storage function* (Hamiltonian) is:

$\mathrm{H}(\mathbf{x}) = \frac{K}{2} \cdot x_{\mathrm{K}}^2 + \frac{i_{\mathrm{nvL}}}{2} \cdot x_{\mathrm{L}}^2 + \frac{i_{\mathrm{nvM}}}{2} \cdot x_{\mathrm{M}}^2$

The gradient of the system's storage function is:

$$\nabla \mathrm{H}(\mathbf{x}) = \begin{pmatrix} g_{\mathrm{xL}} \\ g_{\mathrm{xK}} \\ g_{\mathrm{xM}} \end{pmatrix}$$

$g_{\mathrm{xL}} = i_{\mathrm{nvL}} \cdot x_{\mathrm{L}}$.

$g_{\mathrm{xK}} = K \cdot x_{\mathrm{K}}$.

$g_{\mathrm{xM}} = i_{\mathrm{nvM}} \cdot x_{\mathrm{M}}$.

The Hessian matrix of the storage function is:

$$\triangle \mathrm{H}(\mathbf{x}) = \begin{pmatrix} i_{\mathrm{nvL}} & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & i_{\mathrm{nvM}} \end{pmatrix}$$

The Hessian matrix of the linear part of the storage function is:

$$\mathbf{Q} = \begin{pmatrix} i_{\mathrm{nvL}} & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & i_{\mathrm{nvM}} \end{pmatrix}$$

1.7.2 Dissipation

The dissipation function is:

$\mathbf{z}(\mathbf{w}) = \text{Empty}$

The jacobian matrix of the dissipation function is:

$\mathcal{J}_{\mathbf{z}}(\mathbf{w}) = \text{Empty}$

The jacobian matrix of the linear part of the dissipation function is:

$$\mathbf{Z_l} = \begin{pmatrix} R & 0 \\ 0 & A \end{pmatrix}$$

1.8 Structure

The interconnection matrices $\mathbf{M} = \mathbf{J} - \mathbf{R}$ are given below.

1.8.1 M structure

$$\mathbf{M} = \begin{pmatrix} -R & 0 & -B \cdot e^{-x_{\mathrm{K}}^2} & -1 \\ 0 & 0 & 1 & 0 \\ B \cdot e^{-x_{\mathrm{K}}^2} & -1 & -A & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{M_{xx}} = \begin{pmatrix} -R & 0 & -B \cdot e^{-x_{\mathrm{K}}^2} \\ 0 & 0 & 1 \\ B \cdot e^{-x_{\mathrm{K}}^2} & -1 & -A \end{pmatrix}$$

$\mathbf{M_{xw}} = \mathrm{Empty}$

$$\mathbf{M_{xy}} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$\mathbf{M_{xcy}} = \mathrm{Empty}$

$\mathbf{M_{wx}} = \mathrm{Empty}$

$\mathbf{M_{ww}} = \mathrm{Empty}$

$\mathbf{M_{wy}} = \mathrm{Empty}$

$\mathbf{M_{wcy}} = \mathrm{Empty}$

$\mathbf{M_{yx}} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

$\mathbf{M_{yw}} = \mathrm{Empty}$

$\mathbf{M_{yy}} = \mathrm{Zeros}$

$\mathbf{M_{ycy}} = \mathrm{Empty}$

$\mathbf{M_{cyx}} = \mathrm{Empty}$

$\mathbf{M_{cyw}} = \mathrm{Empty}$

$\mathbf{M_{cyy}} = \mathrm{Empty}$

$\mathbf{M_{cycy}} = \text{Empty}$

1.8.2 J structure

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & -1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^2} & -1.0 \\ 0 & 0 & 1.0 & 0 \\ 1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^2} & -1.0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{J_{xx}} = \begin{pmatrix} 0 & 0 & -1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^2} \\ 0 & 0 & 1.0 \\ 1.0 \cdot B \cdot e^{-x_{\mathrm{K}}^2} & -1.0 & 0 \end{pmatrix}$$

$\mathbf{J_{xw}} = \text{Empty}$

$$\mathbf{J_{xy}} = \begin{pmatrix} -1.0 \\ 0 \\ 0 \end{pmatrix}$$

$\mathbf{J_{xcy}} = \text{Empty}$

$\mathbf{J_{wx}} = \text{Empty}$

$\mathbf{J_{ww}} = \text{Empty}$

$\mathbf{J_{wy}} = \text{Empty}$

$\mathbf{J_{wcy}} = \text{Empty}$

$\mathbf{J_{yx}} = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$

$\mathbf{J_{yw}} = \text{Empty}$

$\mathbf{J_{yy}} = \text{Zeros}$

$\mathbf{J_{ycy}} = \text{Empty}$

$\mathbf{J_{cyx}} = \text{Empty}$

$\mathbf{J_{cyw}} = \text{Empty}$

$\mathbf{J_{cyy}} = \text{Empty}$

$\mathbf{J_{cycy}} = \text{Empty}$

1.8.3 R structure

$$\mathbf{R} = \begin{pmatrix} 1.0 \cdot R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 \cdot A & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{R_{xx}} = \begin{pmatrix} 1.0 \cdot R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.0 \cdot A \end{pmatrix}$$

$\mathbf{R_{xw}} = \text{Empty}$

$\mathbf{R_{xy}} = \text{Zeros}$

$\mathbf{R_{xcy}} = \text{Empty}$

$\mathbf{R_{wx}} = \text{Empty}$

$\mathbf{R_{ww}} = \text{Empty}$

$\mathbf{R_{wy}} = \text{Empty}$

$\mathbf{R_{wcy}} = \text{Empty}$

$\mathbf{R_{yx}} = \text{Zeros}$

$\mathbf{R_{yw}} = \text{Empty}$

$\mathbf{R_{yy}} = \text{Zeros}$

$\mathbf{R_{ycy}} = \text{Empty}$

$\mathbf{R_{cyx}} = \text{Empty}$

$\mathbf{R_{cyw}} = \text{Empty}$

$\mathbf{R_{cyy}} = \text{Empty}$

$\mathbf{R_{cycy}} = \text{Empty}$